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LETTER TO THE EDITOR

AC conductance of an interacting quantum dot: single-electron-level spectroscopy

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Abstract. We calculate the AC and DC current through a quantum double-barrier heterostructure taking into account Coulomb interactions in the well. The AC conductance has a periodic dependence on the external field frequency corresponding to the single-electron-level spacing, and as a function of the leads' chemical potential with a period determined by the interaction. These two different periods enable probing of the two characteristic energy scales of the quantum-dot structure.

Transport properties of a quantum dot weakly coupled to external leads [1] have lately been the subject of intense experimental [2–4] and theoretical [5–8] investigation. Experimental results obtained for the DC conductance of narrow-channel systems [2–4] exhibit several striking features, such as periodic conductance oscillations, magnetic field dependence of the conductance peaks, and irregular temperature dependence. The periodic conductance oscillations are the result of the charging energy of the dot [5–7], but in order to explain the magnetic and temperature dependences of the conductance it is important to take into account the interplay between two relevant energy scales in the quantum dot: the charging energy of the dot, and the spacing of the single-electron energy levels. This has recently been pointed out by Meir and co-workers [8] and, independently, by Beenakker [5].

Very recently several experimental groups have been considering performing measurements of the current in narrow-channel devices when an external AC field is also applied [9]. A possible realization of such an experiment is placing the quantum-dot device in a wave guide. Though several theoretical studies for the AC conductance of a narrow channel have been presented [10–14], to the best of our knowledge, none have incorporated the effect of electron–electron (e–e) interactions in the dot.

In this letter we shall study the influence of an external AC electric field on the current through a quantum dot. In our calculations we shall use a Hamiltonian in which the quantum dot weakly coupled to external leads is modelled as a one-dimensional double-barrier quantum well. The *full* electronic spectrum in the well is taken into account, as well as e–e interactions. The external field and the hopping probabilities into the dot are treated to first order in their respective amplitudes. In the absence of an AC field we qualitatively recover the current behaviour predicted

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by Meir and co-workers [8]. We predict an AC conductance which has both a real part which corresponds to the component of the current which is in phase with the external field, and an imaginary part. Each component is influenced in a different way by the Coulomb interaction energy and the single-electron spectra. This dependence gives a very accurate new experimental tool for the measurement of the single-particle energies in an interacting system.

For the typical experimental set-up the chemical potential in the leads ($\mu_{L(R)}$ for the left (right) lead) is determined by applying an external gate voltage over the leads. The DC current is a function of the chemical potential difference $\Delta V = \mu_L - \mu_R$, and the absolute value of the chemical potential μ_L. An additional external AC electric field $E_0 \cos(\omega_0 t)$ is applied. The energy scales resulting from the electronic confinement in the quantum-dot structure have an extremely important role in characterizing the transport properties of the system. The spacing of the single-electron levels is of the order of $\Delta\epsilon_\alpha \simeq 0.05$ meV, for a typical quantum dot [3]. The second energy scale is the charging energy of the dot e^2/C (C is the capacitance of the dot), which is typically an order of magnitude larger then $\Delta\epsilon_\alpha$. These energy scales have also an important influence on the frequency dependence of the AC current. The typical AC frequencies are up to several GHz [9] which correspond to energies of the order of $\Delta\epsilon_\alpha$.

To describe this experimental set-up we shall use the following model Hamiltonian [15, 16]

$$H = H_0 + H_1 + H_2 + H_3 \tag{1}$$

for which

$$H_0 = \sum_k \epsilon_k a_k^\dagger a_k + \sum_p \epsilon_p b_p^\dagger b_p + \sum_\alpha \epsilon_\alpha c_\alpha^\dagger c_\alpha \tag{2a}$$

where ϵ_k (ϵ_p) are the energy levels of the particles in the left (right) lead, and ϵ_α are the discrete single-electron energy levels in the well. The next term in the Hamiltonian corresponds to the tunnelling between the leads and the well, and is given by

$$H_1 = \sum_{k,\alpha} T_{k,\alpha} a_k^\dagger c_\alpha + \sum_{p,\alpha} T_{p,\alpha} b_p^\dagger c_\alpha + \text{HC} \tag{2b}$$

where the hopping amplitudes $T_{k(p),\alpha}$ are determined by the matrix elements of the left (right) barrier potential between the eigenstate in the well ($|\alpha\rangle$) and the state in the corresponding lead ($|k(p)\rangle$). We note that T has no imaginary components. Due to the application of an AC electric field we have an additional term in the Hamiltonian:

$$H_2 = \left[\sum_{k,\alpha} g_{k,\alpha} a_k^\dagger c_\alpha - \sum_{p,\alpha} g_{p,\alpha} b_p^\dagger c_\alpha + \sum_{\alpha,\gamma} g_{\alpha,\gamma} c_\alpha^\dagger c_\gamma\right] E_0 \cos(\omega_0 t) + \text{HC} \tag{2c}$$

where $g_{k(p),\alpha} = \langle k(p)|e \cdot x|\alpha\rangle$ are the matrix elements of the electric potential in the barrier, which has a form of a dipole moment [17]. The minus sign in the second term of H_2 is in order to keep the same notation for the left and right barriers

(for the right barrier the external potential in the calculation of g is $-e \cdot x$). The third term represents transitions between states in the well, where $g_{\alpha,\gamma} = \langle\alpha|e \cdot x|\gamma\rangle$. There are two main reasons for taking the influence of the external AC electric field in the barrier region also. The contribution of the electric field in the well to the tunnelling current is relatively small as long as $\omega_0 < \Delta\epsilon_\alpha$. On the other hand once the external field frequency is of the order of the single-electron level spacing the main contribution to the AC conductance comes from the well's polarizability, as will be discussed later. Therefore, the electric field in the barrier plays the main role in describing the AC tunnelling current. The second reason is that we shall use a Kubo-like formalism [15, 16] for which the perturbation is proportional to the local current density, which is well defined (in this model) only at the barriers. The e–e interactions are considered only in the well [8], and are taken in the following way:

$$H_3 = \frac{e^2}{2C} \sum_{\alpha \neq \beta} c_\alpha^\dagger c_\alpha c_\beta^\dagger c_\beta \tag{2d}$$

corresponding to the usual Coulomb blockade form. The capacitance C of the well is taken to be slowly varying around the Fermi level of the well [5, 8].

The current which flows through the barriers to first order in the electric field is defined as [15, 16]

$$I_{L(R)} = -(+)ei \int_{-\infty}^{t} dt' \langle [\dot{N}_{L(R)}(t), H_1(t') + H_2(t')] \rangle \tag{3}$$

where for the left lead

$$\dot{N}_L(t) = i[N_L(t), H_1(t)] = i \sum_{k,\alpha} [T_{k,\alpha} a_k^\dagger c_\alpha - T_{k,\alpha} c_\alpha^\dagger a_k] \tag{4}$$

is the change of the total charge in the left lead, and $\dot{N}_R$ is similarly defined. In equation (4) we have neglected the term $[N_L(t), H_2(t)]$, since we are interested in the linear response to the external AC field.

The chemical potential in the well μ_W is determined from the condition that the DC current through the system is homogeneous, i.e. $I_L = I_R$ [18]. Following Mahan [16] we define the left barrier Matsubara response function

$$X_{LW}(i\omega_m) = \sum_{k,\alpha} D_{k,\alpha} \frac{1}{\beta} \sum_{ip_n} G_L(k, ip_n) G_W(\alpha, ip_n - i\omega_m) \tag{5}$$

where $\omega_m = 2m\pi/\beta$ (β is the inverse temperature) is the bosonic Matsubara frequency, and $p_n = (2n+1)\pi/\beta$ is the fermionic frequency. $G_{L(W)}$ are the corresponding Matsubara Green functions in the left lead (well). $D_{k,\alpha} = T_{k,\alpha}^2$ for the DC component of the current, and $D_{k,\alpha} = T_{k,\alpha} g_{k,\alpha}$ for the AC part. For the right barrier X_{RW} is similarly defined.

We assume that the electrons in the leads can be considered as free electrons; hence for the left lead $G_L(k, ip_n) = (ip_n - \xi_k)^{-1}$, $\xi_k = \epsilon_k - \mu_L$. The effect of e–e interactions in the well is considered by taking the first-order corrections to the electron's self-energy Σ_N, therefore $G_W(\alpha, ip_n) = (ip_n - \xi_\alpha - \Sigma_N)^{-1}$. The dominant

contribution to Σ_N is from the direct term (the existence of a net negative charge in the well due to the additional electrons that tunnel into the well, which is the well known Coulomb blockade [1]). Under these assumptions $\Sigma_N = Ne^2/C$; N is the number of excess electrons in the well.

Using our previous definitions for the current (equation (3)), one obtains the following result for the DC part of the current:

$$I_L^{dc} = -2e \operatorname{Im} X_{LW}^{ret}(\mu_W - \mu_L) \tag{6}$$

which is equal to I_R^{dc} as follows from our definition of μ_W.

The most interesting behaviour is encountered when the chemical potential difference is smaller than the energy cost of adding an additional electron to the well. In this case for low temperatures (compared with the single-electron level spacing) one can easily deduce that μ_W follows the right lead chemical potential μ_R as long as there is no interacting electronic level in the range ΔV above μ_R. Once such a level enters this range μ_W joins it. In the high-temperature limit μ_W is simply in the middle of ΔV.

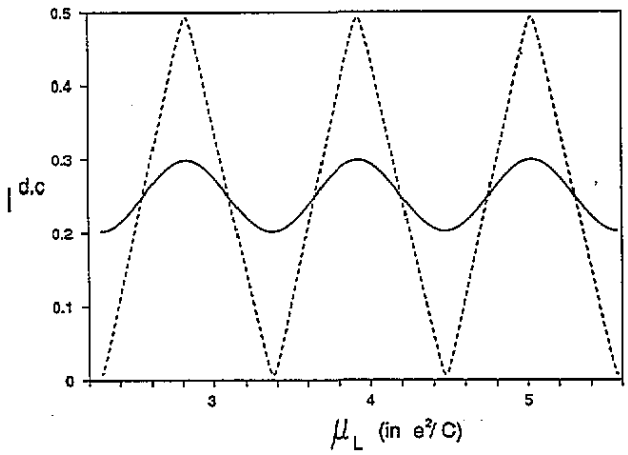

Figure 1. The DC current (in arbitrary units) as a function of the chemical potential of the left lead μ_L. The full curve corresponds to a temperature higher than the single-electron level spacing, $1/\beta = 0.2(e^2/C)$. The broken curve represents a temperature lower than this level spacing, $1/\beta = 0.05(e^2/C)$.

To illustrate the features of the DC current we calculated it for the following numerical values of parameters: $\epsilon_\alpha = 0.1(e^2/C)\alpha$ (where α is an integer), which is similar to the expected experimental ratio between the single-electron level spacing and the charging energy. The chemical potential difference between the leads $\Delta V = 0.05(e^2/C)$ is kept constant. For simplicity we assumed a constant density of states in the leads, and hopping amplitudes T. In figure 1 we plot the DC part of the current I^{dc} as a function of μ_L for $1/\beta = 0.2(e^2/C)$, and $1/\beta = 0.05(e^2/C)$. One notices that the DC current exhibits oscillations with a period corresponding to $e^2/C + \Delta\epsilon_\alpha$, where $\Delta\epsilon_\alpha$ is the single-electron level spacing in the well. For intermediate temperatures $\Delta\epsilon_\alpha < 1/\beta < e^2/C$, the oscillations remain although the

peaks are broadened. We can also see the irregular temperature behaviour [8] by taking variable hopping amplitudes, and density of states in the leads.

We now turn to the calculation of the AC conductance. As is customary for AC currents, we may write I^{ac} as composed of two parts:

$$I^{ac} = AE_0 \cos(\omega_0 t) + BE_0 \sin(\omega_0 t). \tag{7}$$

We can define an impedance Z for the structure, such that $A = e\,\mathrm{Re}\,Z$, and $B = -e\,\mathrm{Im}\,Z$, where according to the Ramo–Shockley theorem [12, 19] $Z = (Z_L + Z_R)/2$. In our case

$$Z_L = \mathrm{i}[X^{ret}_{LW}(\mu_W - \mu_L + \omega_0) - X^{adv}_{LW}(\mu_W - \mu_L - \omega_0)]. \tag{8}$$

Z_R is calculated in a similar fashion. Using the definition of X in equation (5) we obtain the following result for the real part of Z_L:

$$\begin{aligned}\mathrm{Re}\,Z_L = \frac{1}{2}\sum_\alpha \int \mathrm{d}\epsilon_k R(\epsilon_k) T_{k,\alpha} g_{k,\alpha}[n_F(\xi_k) - n_F(\xi_\alpha + \Sigma_N)] \\ \times \{\delta(\mu_L - \mu_W + \epsilon_\alpha + \Sigma_N - \omega_0 - \epsilon_k) + \delta(\mu_L - \mu_W + \epsilon_\alpha \\ + \Sigma_N + \omega_0 - \epsilon_k)\}\end{aligned} \tag{9}$$

where $R(\epsilon_k)$ is the density of states in the left lead. The imaginary part of Z_L is

$$\begin{aligned}\mathrm{Im}\,Z_L = \frac{1}{2\pi}\sum_\alpha \int \mathrm{d}\epsilon_k R(\epsilon_k) T_{k,\alpha} g_{k,\alpha}[n_F(\xi_k) - n_F(\xi_\alpha + \Sigma_N)] \\ \times \left\{\frac{1}{\mu_L - \mu_W + \epsilon_\alpha + \Sigma_N - \omega_0 - \epsilon_k} \right. \\ \left. - \frac{1}{\mu_L - \mu_W + \epsilon_\alpha + \Sigma_N + \omega_0 - \epsilon_k}\right\}.\end{aligned} \tag{10}$$

Z_R is derived in a corresponding way. Using the above results it is possible to calculate an explicit form for the two components of the AC current A and B appearing in equation (7).

To show the main features of both components we calculated them for the same parameter values as used for the DC case. We plot the results for $1/\beta = 0.2(e^2/C)$. Following the assumptions adopted for the DC calculation we consider a constant hopping amplitude g. In figure 2 we plot coefficient A as a function of μ_L for different frequencies ω_0.

One may immediately observe that A for any ω_0 has the underlying period of $e^2/C + \Delta\epsilon_\alpha$. The amplitude of A shows a very interesting additional periodic dependence on the frequency. This new period is exactly $\Delta\epsilon_\alpha$. It is instructive to note that in addition to the information obtained through the DC conductance measurements, the real component of the AC conductance gives us a very precise probe into two different energy scales, effectively enabling us to perform single-electron-level spectroscopy. Even for low frequencies it is possible to deduce the spacing of the single-electron levels by following the amplitude changes of A, as a function of the frequency. The role of temperature is similar to the DC case.

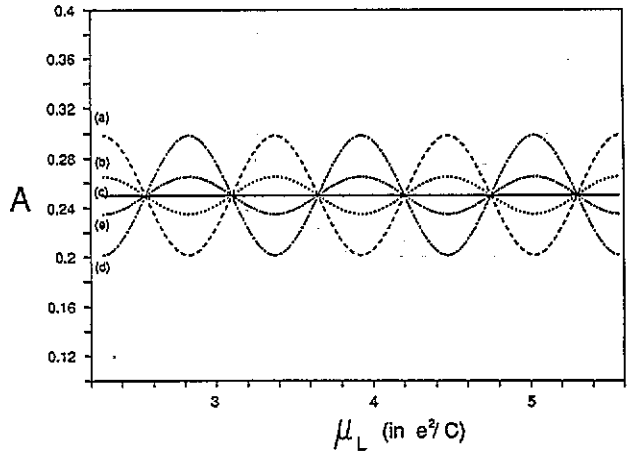

Figure 2. The real part of the AC conductance A (in arbitrary units) as a function of μ_L for different external frequencies ω_0: curve a, $\omega_0 = 0$; curve b, $\omega_0 = 0.02$; curve c, $\omega_0 = 0.025, 0.075$; curve d, $\omega_0 = 0.05$; curve e, $\omega_0 = 0.07$, where ω_0 is given in units of e^2/C. As is described in the text the behaviour of A is periodic as a function of ω_0, giving $A(\omega_0) = A(\omega_0 + \Delta\epsilon_\alpha)$. For this plot $\Delta\epsilon_\alpha = 0.1$.

In figure 3 we present the dependence of B on μ_L for different frequencies. Again, the periodicity of $e^2/C + \Delta\epsilon_\alpha$ is observed. The frequency changes the positions of the peaks as a function of μ_L, while the zeros remain at the same locations. As in the previous case, the spacing of the single-electron levels determines the period of those shifts. One should note that although A is non-zero even for $\omega_0 = 0$, the amplitude of B vanishes for that case.

In all the above discussion we have assumed that the electric field has an important contribution only if we consider its influence on the barrier regime. This is correct as long as the frequency ω_0 is smaller than the spacing of the single-electron levels in the well. Once the frequency is of order $\Delta\epsilon_\alpha$, the contribution to the well polarizability due to oscillations of the electrons in the well becomes important [20]. This phenomenon affects the AC conductance although no electrons are transferred through the barriers. The well polarizability will cause sharp peaks of width T in the real part of the conductance as a function of frequency, at values corresponding to $\omega_0 = \epsilon_\alpha - \epsilon_{\alpha'}$, which may also be used as an accurate way to observe the single-electron spectra. The width of these additional peaks is small compared to $\Delta\epsilon_\alpha$, as must be assumed in order to obtain resonant energy levels in the well. Therefore, these additional peaks, which will appear only at the above mentioned discrete frequencies, will not wash out the previous discussed periodic behaviour even for $\omega_0 > \Delta\epsilon_\alpha$.

It is essential at this point to re-evaluate the assumption concerning the existence of thermal equilibrium in the well. To obtain equilibrium the hopping rate T^2 should be lower than the thermalization rate (which is due to various scattering mechanisms in the well). This condition should be verified for the DC conductance. For real systems it was estimated [21] that the two rates are of the same order of magnitude. Therefore, we can expect to see some differences in the details of the peak shapes, but the qualitative behaviour will not change. In the linear response regime, the hopping rate for the AC current is proportional to gTE_0, which for $E_0 < T/g$ should not

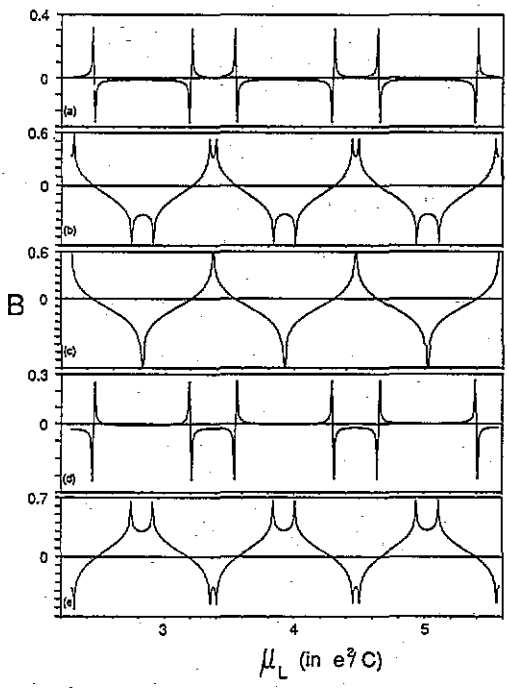

Figure 3. The imaginary part of the AC conductance B (in the same arbitrary units as A), for different external frequencies ω_0: (*a*) $\omega_0 = 0.001$, (*b*) $\omega_0 = 0.02$, (*c*) $\omega_0 = 0.025$, (*d*) $\omega_0 = 0.05$, (*e*) $\omega_0 = 0.07$. The periodicity in e^2/C for each frequency is evident. An additional periodic dependence on the frequency is observed regarding the locations of the peaks as a function of μ_L. It is important to note that the amplitude of the peaks increases for $\omega_0 \to \omega_0 + \Delta\epsilon_\alpha$.

disturb the conditions for thermal equilibrium.

Another point of interest is non-linear effects. For example, an additional frequency scale has been experimentally observed for one-dimensional arrays of tunnel junctions [22]. In this paper, which considers the conductance of a quantum dot, only the linear regime was treated. A full quantum treatment of the non-linear regime would be of much interest.

In conclusion, we have presented a description of the conductance of a double-barrier structure, taking into account a multiple-level well with e–e interactions, finite temperature, DC bias, and an AC external field. We found that the AC conductance is composed of two parts, the real part which represents photon assisted hopping processes, and the imaginary part corresponding to the level broadening due to the external field. Both exhibit periodic behaviour in μ_L and ω_0, with periods corresponding to the two different energy scales in the well.

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